

■ Prove the following

[A] $|ab| = |a||b|$

[B] $\frac{a}{b} = \frac{|a|}{|b|}$

[1] $|x| < |y| \iff x^2 < y^2$

[2] $||x| - |y|| \leq |x + y|$

[3] $||x| - |y|| \leq |x - y|$

[4] $|x - y| \leq |x - z| + |z - y|$

[5] $|x + y| + |x - y| \geq |x| + |y|$

[6] $a^2 + b^2 + c^2 \geq ab + bc + ac$

[7] $x^2 + 4y^2 + 1 \geq 2xy + x + 2y$

[8] $(a^2 + b^2)(c^2 + d^2) \geq (ac + bd)^2$

■ Proofs